

Double Trace Deformations, Infinite Extra Dimensions, and Supersymmetry Breaking

Micha Berkooz¹

Department of Particle Physics, The Weizmann Institute of Science,
Rehovot 76100, Israel

It was recently shown how to break supersymmetry in certain AdS_3 spaces, without destabilizing the background, by using a “double trace” deformation which localizes on the boundary of space-time. By viewing spatial sections of AdS_3 as a compactification space, one can convert this into a SUSY breaking mechanism which exists uniformly throughout a large 3+1 dimensional space-time, without generating any dangerous tadpoles. This is a generalization of a Visser type infinite extra dimensions compactification. Although the model is not Lorentz invariant, the dispersion relation is relativistic at high enough momenta, and it can be arranged such that at the same kinematical regime the energy difference between between former members of a SUSY multiplet is large.

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¹ Incumbent of the Recanati career development chair of energy research. E-mail : Micha.Berkooz@weizmann.ac.il.

1. Introduction

One of the main puzzles facing String theory is that of the cosmological constant. Although several possible approaches have been proposed ([1], [2], [3], [4], [5], [6], [7], [8], [9] and others), no calculable and natural idea has been demonstrated convincingly (at least yet, as some of the approaches are currently being investigated). The problem appears in vacua without supersymmetry, in which one often runs into the related problem of moduli stabilization. It may be that we just did not find yet the correct vacuum of String theory, or it may be that we are missing a key conceptual piece of the puzzle. It is therefore worth exploring new mechanisms that might be relevant to this puzzle, even if at their preliminary stages they are not phenomenologically viable.

One class of attempts to solve the cosmological constant problem relies on infinite extra dimensions compactifications [7][10][11][12]. In this paper we will discuss such a compactification based on AdS_3 , and explore its relation to double trace deformations² [14] and SUSY breaking [15]. This will give a new way in which infinite extra dimensions might help solve the cosmological constant problem. Double trace deformations can be used to change the boundary action and boundary conditions [16][17][18] on fields in the non-compact directions transverse to our world, and SUSY breaking will occur when we will introduce SUSY breaking boundary conditions. However, only in very special spaces will the change of boundary conditions influence the physics, and we will focus on one such case.

The example will be (primarily) the familiar $AdS_3 \times S^3 \times T^4$ background of string theory, which is dual [19][20][21] (for a review see [22]) to a 1+1 conformal field theory on a moduli space of Instantons. To make this into an infinite extra dimension compactification we will pick a T^3 out of the T^4 , and take it to be of very large radius relative to the radius of curvature of AdS_3 (which we will keep finite, although smaller than the string scale). The remaining circle from the T^4 will be taken to be small. The time direction of the AdS_3 and the 3 large coordinates of the T^3 can then be considered as large dimensions, and the rest as “compactification dimensions”. The modes that we will be interested in are the normalizable modes on AdS_3 . In terms of relation to previous work on large extra dimension, this model is a generalization of [11].

Our main interest will be in the new SUSY breaking mechanism described in [15],

² For previous work on double trace operators in the context of the AdS/CFT correspondence see for example [13].

which uses a double trace deformation [14] of the CFT to break SUSY and can be shown to not destabilize the vacuum, even at the non-perturbative level. In the low energy effective action, multi trace deformations localize to the boundary of space [16][17][18], and since the boundary of space-time is parallel to $R^1 \times T^3$, we get uniform SUSY breaking throughout space-time. Generally speaking multi-trace deformations permit the introduction of new parameters into String theory, which can be used to change the effective action. But contrary to one of the dogmas of String theory, these parameters are not given by expectation values of scalar fields, alleviating the generic Dine-Seiberg problem of dynamical SUSY breaking in String theory.

The outline of the paper is the following. In section 2 we summarize the model and discuss some algebraic aspects of infinite extra dimension compactification. The main new results of the paper are in sections 3 and 4. In section 3 we discuss supersymmetry breaking. We start with a supersymmetric model and deform it to a new theory where for a class of particles, in a kinematical regime where their dispersion relation is approximately relativistic, the splitting between between former members of a SUSY multiplet is large. In section 4 we point out, based on some known facts about quantization of $SL(2, R)$, some novel features of the 3+1 dimensional effective action. In section 5 we discuss generalizations of the model, which might help remedy some of the phenomenological difficulties.

2. Summary of the Model

2.1. The Model

The basic example that we will explore is $AdS_3 \times S^3 \times T^4$. We will single out 3 circles out of the T^4 and take them to be very large. We will view these 3 circles + the time direction in global AdS_3 as our large space and we will consider the rest as a compactification. The remaining circle of the T^4 will be taken to be small.

We will denote the coordinates of the large T^3 inside T^4 by $x^1..x^3$, and its volume by v_3 . The 4th circle will be denoted by x^4 and will satisfy $x^4 \sim x^4 + R_4$. The relevant formulas for the background are the (following the notations of [23])

$$\frac{1}{g_s^2} = \frac{p}{v_3 R_4 k} \tag{2.1}$$

$$ds^2 = k \frac{r^2}{l_s^2} d\gamma d\bar{\gamma} + k l_s^2 \left(\frac{1}{r^2} dr^2 + d\Omega_3^2 \right) + dx_i dx^i + dx_4 dx^4$$

$$H_0 = 2k(\epsilon_3 + *_6\epsilon_3),$$

where g_s is the 10D string coupling, k is the level of the $SL(2, R)$ and of the $SU(2)_{WZW}$, p is a positive integer, H_0 is the NS-NS 3-form field strength, ϵ is the volume form on AdS_3 , and $*_6\epsilon$ is the volume form on the S^3 . The background is the near horizon limit p fundamental strings and k NS 5-branes.

The regime of parameters that we are interested in is

1.

$$k \gg 1 \tag{2.2}$$

In this paper we would like to have a gap between the ‘‘Long string modes’’ ([24][25][26]) and the gravity fields, since we would like to concentrate on the discrete spectrum. It may be that one can relax this condition, since the CFT at finite k is under some control.

2.

$$v_3/l_s^3 \gg \sqrt{k} \tag{2.3}$$

This is a condition that the momentum modes in the T^3 direction will be more finely spaced than the discrete modes in the spatial slices of AdS_3 , such that we will view the T^3 as the large dimensions, and spatial slices of AdS_3 (and the S^3) as a compactification.

3.

$$g_s \ll 1 \tag{2.4}$$

4.

$$R_4 \sim l_s \tag{2.5}$$

such that we can regard the 4th circle of the T^4 as a compactification.

Condition 2 is to be taken with a grain of salt since the compactification is not Lorentz invariant. It guarantees that momentum modes in the T^3 are more finely spaced than in the AdS direction, but all of this is on top of energies whose scale is set by the AdS scale. One can either accept this violation of Lorentz invariance, or we can discuss momenta larger than the AdS scale. In this case there would be many modes below our kinematical scale, but this is no worse than most infinite extra dimension compactifications. We will use this kinematical regime in section 3 when discussing SUSY breaking, and we will comment in section 5 on how one might proceed to improve Lorentz invariance.

The fields that we are interested in - the “stuff” that makes out matter in our world - are the normalizable modes of global AdS_3 . These have a discrete physical spectrum with the dispersion relation³

$$w = \Lambda(1 + n_L + n_R) + \sqrt{\Lambda^2 + m^2 + q^2} \quad (2.6)$$

where m is the 6 dimensional mass of the particle on $AdS_3 \times T^3$, q is the 3D momentum (in T^3) and Λ is the scale set by AdS_3 . As happens in many infinite extra dimension compactifications⁴ the spectrum is spaced with some spacing set by a cosmological parameter [30][31] (in other infinite extra dimension compactification it is a continuum).

One may complain that the choice of a time direction in AdS_3 is not unique because of the conformal invariance of AdS . But a 3+1 dimensional observer will report the same fact as the result of a spectrum generating algebra (the space-time conformal symmetry of the model). Alternatively, in more general cases, one can try to break conformal invariance by a relevant operator. In this case there might be a preferred time direction (and the spectrum generating algebra would only be asymptotic at high energies).

2.2. Properties of the Model

Let us summarize some properties of the model (some were already mentioned before):

1. The space has a translationally invariant large $R^{1,3}$ component. At length scales much smaller than the radii of T^3 , it also has an approximate $SO(3)$ symmetry.
2. The 3+1 theory has a spectrum generating algebra, which is the space-time conformal symmetry, and a known dual which allows us non-perturbative control over the dynamics.
3. Novel features of the action: as discussed before, it has proven difficult to address the cosmological constant within the ordinary rules of low energy effective action (i.e a Lagrangian with up to 2 derivatives, analytic in the fields and their derivatives at low momentum). It is therefore interesting to explore what more exotic effective actions

³ The choice of sign before the $\sqrt{}$ can be changed in some cases [27]. In fact it has to be changed in some SUSY cases. We will focus on the simpler case here, which is valid for large enough m .

⁴ and in some supergravity models that have a dS vacuum [28][29]

can appear in String theory. We will see in section 4 that we obtain a rather peculiar effective action already at the level of the free action.

4. The background is not Lorentz Invariant: The dispersion relation is given in (2.6). However, we can go to high momentum relative to Λ where it would appear to be relativistic again (but in this regime we probe the higher dimensional space). We will see, however, that this regime is interesting for the purposes of SUSY breaking. We will also comment on how to try to improve the situation in section 5.

More generally, given that we have not been very successful in explaining the cosmological constant so far, perhaps it is worthwhile giving up some current principles such as Lorentz invariance at ultra low energies⁵. For example, in the case we are discussing here, the non Lorentz invariant dispersion relation (2.6) removes the zero energy mode of any scalar field. It therefore restricts the extent to which quantum corrections can destabilize the vacuum.

5. A new SUSY breaking mechanism, discussed in section 3, which does not destabilize the vacuum: even though the physical normalizable modes on AdS_3 do not contain a zero energy mode for any field, it is still rather difficult to find a non-supersymmetric stable $AdS_3 \times M$ background. The reason is that scalar fields that correspond to marginal operators in the dual theory can develop tadpoles, uniformly throughout AdS , that cannot be compensated in a way that preserves conformal invariance and they typically destabilize the vacuum [33].
6. The spectrum does not possess any zero energy states⁶ (in particular, a massless graviton): Since the energy eigenvalues are related to the dimension of the operator on the dual theory, it is clear that we cannot have any zero modes in this theory - this would correspond to a dimension 0 operator (A compactification with no physical zero energy modes is interesting in itself).

It is worth noting two things. The first is that the sizes of the T^3 are moduli of the CFT. Hence, they correspond to mass zero in formula (2.6). This implies that there are low energy symmetric 2-tensor fields. Another point is that modes of the gravitons

⁵ Another approach to the cosmological constant problem using Lorentz symmetry breaking appears in [32]

⁶ A different way of modifying the low energy dispersion relation in String theory was recently suggested in [34], and in low energy effective action in [35]. The relation between these discussions and the mechanism discussed in this paper remains to be elaborated

with polarization in directions of the T^3 are scalar operators from the point of view of the dual CFT. Since the only bound on the dimension of scalar operators in 1+1 dimensions is that it would be larger than zero (both in the non-supersymmetric and the supersymmetric cases), it is perhaps possible to find a generalization of the model which will contain a graviton with very low energy modes.

2.3. Relation to previous work

This compactification might be considered as an “infinite extra dimension” compactification since the proper distance along a spatial section ($t=\text{const}$) of the AdS_3 from its center to the boundary is infinite. Despite this fact, one obtains a discrete on-shell spectrum, as is standard in the AdS/CFT correspondence, due to the warp factor (see for example [36], and references therein).

From existing types of infinite extra dimension compactifications, the model discussed here resembles most the examples of [11], and it is different from the more familiar infinite extra dimension compactifications of the Randall-Sundrum (2) type ([7][10][12]). In the compactification in this paper we borrow only the time direction of AdS to be part of the “non-compact” directions (and 4 dimensionality is achieved using coordinates transverse to the AdS), whereas there all the dimensions of space-time are from the AdS directions. Furthermore, in the RS2 type compactification one usually has a brane and horizons at its two sides. The tension of the brane usually has to be fine-tuned in order to have a 4 dimensional Minkowski space, and therefore its stringy origin remains unclear⁷. In our case, the brane is replaced by the central region of global AdS_3 , where normalizable states localize, and the boundaries of the space are the boundaries of AdS itself (rather than horizons). The model clearly exists within String theory (with the addition of a full non-perturbative definition).

Another class of “infinite extra dimension” compactifications is one in which the brane is embedded in a space which at infinity away from the brane approaches flat space. One then tries to localize a graviton on the brane (for example [8][39]). In these models the localization of the graviton is usually achieved by some fine tuning at the region of the brane, whereas in our model the localization of excitations is due to the behavior of space at infinity.

⁷ A realization of the RS scenario in String theory was suggested in [37]. In [38] it was shown that indeed one can solve the hierarchy problem this way, but the issue of stability of the background in this realization remains problematic.

2.4. Infinite extra dimensions and Spectrum Generating Algebras

The reason that the supersymmetry breaking mechanism of [15] does not destabilize the background is the 1+1 dimensional conformal symmetry of the background. A 3+1 observer will explain it as a result of his world possessing a spectrum generating algebra. We would like to argue that quite generally a spectrum generating algebra might be a natural thing to explore in the context of infinite extra dimension compactifications.

It is usually suggested that infinite extra dimension compactifications might help solve the cosmological constant problem by having corrections to the energy of a brane, or otherwise a loci where excitations are trapped, change the geometry away from it, rather than bending the brane itself⁸. However, new algebraic structures might be another important motivation, and in themselves might help to solve the cosmological constant problem, or decouple states in some processes etc. A non-trivial spectrum generating algebra is one example - ie, the Hamiltonian of the theory is part of a more complicated algebra⁹ which contains operators which do not commute with it¹⁰. In such cases the dynamics of excitations at various energy scales would be related to each other, and in particular their contributions to the cosmological constant would be correlated. This might be a way towards solving the cosmological constant problem.

Consider for example a warped compactification

$$ds^2 = f(y)\eta_{\mu\nu}dx^\mu dx^\nu + g_{ij}dy^i dy^j \quad (2.7)$$

where $\mu, \nu = 0..3$ and $i, j = 4, ..9$ (if we want to restrict ourselves to starting from 10 dimensions). Suppose the manifold has a discrete symmetry which leaves the metric invariant but rescales f by some number c , i.e.

$$y = y(\hat{y}), \quad \hat{g} = g, \quad \hat{f}(y) = cf(y).$$

Note that there is no contradiction with Einstein's equations - if there is a solution to Einstein's equations, then by rescaling x there exists a solution where f is rescaled. A

⁸ The problem is then to establish the existence of an effective 4D graviton, and to deal with the continuum of states which often follows from the infinite extra dimensions

⁹ Lorentz symmetry is of course such an algebra, but its representation structure isn't rich enough to constrain the cosmological constant.

¹⁰ Although we will not require the entire spectrum to be in a single representation of this algebra. Hence, it does not "generate" the entire spectrum

concrete example is the Poincare patch of AdS_p , but we would like to consider the more general case.

The action of a scalar field in this background is

$$\int d^4x d^6y \sqrt{g} f \left(\eta^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) + f g^{ij} \partial_i \phi \partial_j \phi + f V(\phi) \right)$$

Under the transformation $y = y(\hat{y})$, the action becomes

$$\int d^4x d^6y \sqrt{g} c f \left(\eta^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) + c f g^{ij} (\partial_i \phi) (\partial_j \phi) + c f V(\phi) \right)$$

If we append this transformation by

$$x_{new} = c^{\frac{1}{2}} x$$

we return to the same Lagrangian. If the 3+1 dimensional theory does have a discrete spectrum of masses, this transformation will have the effect of multiplying the masses of particles by c . Hence warped compactifications have a chance of giving rise to a string vacuum with interesting algebraic properties in the mass spectrum.

It is very restrictive for a manifold to have such a symmetry, which is how infinite extra dimensions come in - it is easy to show that no such symmetry exists in the case that the internal manifold is compact. In fact, in a Lorentz invariant theory if we assume that the spectrum of masses is bounded from below by a finite gap above zero, then we clearly cannot have a symmetry which acts by simply multiplying the masses. However, more complicated algebraic structures are not excluded, such as in the case that we are discussing.

This discussion was in a Lorentz invariant theory (and in non-Lorentz invariant cases, one expects an even richer structure). It is not clear however whether spectrum generating algebras (other than Lorentz or conformal symmetry) can be incorporated in a Lorentz invariant theory at all, or in a theory on a dS space (which is favored observationally). But we can take an AdS space and see whether we can get a model with at least 3+1 large dimensions (although with no Lorentz symmetry), which is our model.

3. Double trace deformations and SUSY breaking

The interest in a compactification of this type is twofold. The first is that it is an infinite extra dimension compactification with a discrete spectrum. The other is that,

by extending the framework of String theory to allow non-local string theories [14], it introduces new parameters which can be used to modify the effective action. They do so by modifying the boundary behavior of the theory, but due to the geometry of the compactification, these modifications are translationally invariant in $R^{1,3}$. In particular we can use such deformations to break supersymmetry without destabilizing the vacuum.

Finding a stable non-supersymmetric vacuum is usually complicated in String theory because, among other problems, tadpoles will be generated for some fields, at some order in the genus expansion. In the context of the AdS/CFT correspondence, tadpoles are dangerous only for fields which correspond to marginal operators [33] (and a tadpole can be generated only for a field which is invariant under all the symmetries of the model). As was pointed out in [15], it is therefore remarkable that one can find a SUSY breaking deformation of the model which does not destabilize the vacuum. In the following we will briefly review [15], and proceed to see how it will vary the 3+1 dimensional spectrum. In particular we will see that one can be in a regime where the splitting between the fermionic and bosonic members of the same former SUSY multiplet is arbitrarily large.

Of course, since we do not have Lorentz invariance, supersymmetry is not 3+1 dimensional. Rather it closes only on time translations. Furthermore, since we are in the NS sector of the boundary theory, the supersymmetries are half integer moded, and the $G_{\pm\frac{1}{2}}$ do not commute with the Hamiltonian. When we say that we break supersymmetry we mean that we break even this algebra, and that the splitting of energies will be much larger than the $\Lambda/2$ set by the structure of the NS-sector. At the same time we can work in a regime where $q \gg \Lambda, \Lambda(1 + n_L + n_R)$ in (2.6), where the dispersion relation will be approximately Lorentz invariant¹¹.

Let us review some properties of the supersymmetry of the model. The model has an $SU(2)_L \times SU(2)_R$ R-symmetry. We pick a $U(1)_L \times U(1)_R$ subgroup of the R-symmetry group, which we will denote as $J(z)$ and $\tilde{J}(\bar{z})$, and we will view the theory as a (2, 2) theory [40] (z and \bar{z} are coordinates in the boundary CFT). We can bosonize the currents as $J = i\sqrt{c/3}\partial\eta$, $\tilde{J} = i\sqrt{c/3}\bar{\partial}\bar{\eta}$, where c is the central charge of the theory, which in our case is of order k (see (2.1)), and $\eta, \bar{\eta}$ are canonically normalized scalar fields. We then

¹¹ This is not much worse than most attempts at an infinite extra dimension where there are typically many excitations, if not a continuum, below the physically relevant range of momenta and masses.

decompose the operators in the theory as

$$O = e^{ip\eta + i\tilde{p}\tilde{\eta}} P(\partial^n \eta, \bar{\partial}^m \bar{\eta}) \hat{O} \quad (3.1)$$

where P is a polynomial and \hat{O} is an operator in the CFT/ $U(1)$. Under this decomposition the supercharges have charges

$$(p, \tilde{p}) = (\pm\sqrt{3/c}, 0), \quad (p, \tilde{p}) = (0, \pm 3/c), \quad (3.2)$$

and we will denote $3/c$ by Δ . So far this was general $(2, 2)$ supersymmetry. In the specific case of $AdS_3 \times S^3 \times T^4$, $c \sim k = Q_1 Q_5$ and the spectrum of charges

$$p \sim \frac{q}{\sqrt{c}}, \quad \tilde{p} \sim \frac{\tilde{q}}{\sqrt{c}} \quad (3.3)$$

where q and \tilde{q} are weights of $SU(2)$.

To visualize what follows one should think about the left and right $U(1)$ symmetries as roughly arising from a single circle of radius approximately \sqrt{k} , giving rise to the spacing of charges above (although in the full theory we may not see all the spectrum of the circle - we may need to do some additional projections, and correlate the charges (p, \tilde{p}) with the operator \hat{O} . But the analogy with the circle is enough for the purposes of scaling). This is also clear in space-time - each $U(1)$ acts as an isometry inside the S^3 whose radius is proportional to \sqrt{k} . In this picture the supercharges have both momentum and winding around this circle.

The susy breaking double trace [15] deformation is then the addition of

$$\mathcal{S} \rightarrow \mathcal{S} + \tilde{h} \int d^2 z J(z) \tilde{J}(\bar{z}) \quad (3.4)$$

to the space-time CFT (and on the GR side to boundary action) with arbitrary coefficient \tilde{h} (more details can be found in [15]). This deformation amounts to changing the radius of the $U(1)$ circle¹². If we have an operator with (p, \tilde{p}) then under this deformation (and keeping $(\eta, \tilde{\eta})$ canonically normalized) the momenta change as

$$\begin{pmatrix} p' \\ \tilde{p}' \end{pmatrix} = \begin{pmatrix} \cosh^2(\gamma) & \sinh^2(\gamma) \\ \sinh^2(\gamma) & \cosh^2(\gamma) \end{pmatrix} \begin{pmatrix} p \\ \tilde{p} \end{pmatrix}, \quad (3.5)$$

¹² One should emphasize that we are not squashing the S^3 . That would correspond to condensing on the boundary an operator that corresponds to a metric field. Here we are condensing a double trace operator which corresponds to a pair of gauge fields in the bulk.

where γ is a function of \tilde{h} ($\gamma = 0 \leftrightarrow \tilde{h} = 0$). Hence the dimensions of charged operators shift as we perform the double trace deformation. In particular, the dimension of the supercharges will shift, which means that we have broken supersymmetry. Since this deformation can be shown to be truly marginal we did not destabilize the background.

We would like to see whether we get a large splitting in the 3+1 dimensional multiplets. The dimension of the operator O (3.1) receives a contribution from $e^{ip\eta+i\tilde{p}\tilde{\eta}}$ and from $P\hat{O}$. The total dimension is $1 + \sqrt{1 + m^2 + q^2}$ (dimensions are corrected using Λ , and q is the 3 dimensional momentum), where m receives contributions both from the mass in 10 dimensions and the angular dependence on $S^3 \times S^1$ (part of which are the $U(1)$ R-charges). By subtracting the contribution of the exponential we deduce that the contribution of $P\hat{O}$ is

$$1 + n_L + n_R + \sqrt{1 + m^2 + q^2} - \frac{3(p^2 + \tilde{p}^2)}{2c}. \quad (3.6)$$

As we deform the theory, the contribution from the exponentials in (3.1) change. As a function of γ the total dimension is

$$1 + n_L + n_R + \sqrt{1 + m^2 + q^2} + \frac{3(p^2(\gamma) + \tilde{p}^2(\gamma) - p^2 - \tilde{p}^2)}{2c}, \quad (3.7)$$

where from (3.5)

$$p^2(\gamma) + \tilde{p}^2(\gamma) = \frac{1}{2} \left(e^{2\gamma} (p + \tilde{p})^2 + e^{-2\gamma} (p - \tilde{p})^2 \right). \quad (3.8)$$

Consider now taking $\gamma \rightarrow -\infty$. In this case, for a state not to decouple from the spectrum - i.e., for its energy in 3+1 dimensions not to go to infinity - it needs to satisfy

$$p = \tilde{p}. \quad (3.9)$$

However, if this is true for some given state, then it cannot be true for its partners in the multiplet under an odd number of applications of the supercharges. The latter will satisfy $p = \tilde{p} \pm (2n + 1) * \Delta$ for some integer n , and hence will decouple from the 3+1 spectrum (the dimension formula for a fermion is slightly different than (2.6) and (3.6), but the (p, \tilde{p}) dependence is the same). I.e, we concentrate on momentum modes along the circle of the left and right $U(1)$ when we take its radius to infinity. The supercharge have non-zero winding number in this convention and hence so do some of the SUSY partners of the momentum modes. The latter, however, are now lifted to infinity as we take the radius of the circle to infinity. If the states that correspond to momentum modes ($p = \tilde{p}$)

are bosons then we certainly lift all the fermions in the former multiplet to high energy (as well as some of the bosons), and vice versa if the momentum modes are fermions.

Furthermore, all of this can happen in a regime where $q \gg m, 1, p^2/c, \tilde{p}^2/c$, where the particle will have an approximately Lorentz invariant dispersion relation.

4. The 3+1 Dimensional Point of View

In this section we will discuss what the quadratic action for a scalar field on this space looks like, from the point of view of the 3+1 dimensional observer. We will begin with a scalar field of some mass m on $AdS_3 \times T^3$, and KK reduce to $R \times T^3$. By the action we mean the off-shell quadratic action for this field, which is well defined since we are discussing a scalar field in a fixed background. To compute the action we need to compute the eigenvalues of the Laplacian on this space, and how they depend on the 3+1 momenta. This action that we will get will turn out to be a generalized free field theory with some unusual features.

In general, in warped compactifications, one expects a generalized free field theory already at the quadratic level when performing a Kaluza-Klein reduction. Using the metric as in (2.7) we insert

$$\phi(x, y) = \Sigma_n \int d^4k e^{ikx} \phi_{n,k}(y)$$

where x are coordinates on $R^{1,3}$, y are the rest of the coordinates and $\phi_{n,k}$ is a complete set of functions of y (for a given k) which we will specify momentarily. The action then becomes

$$\int d^4k \Sigma_n \int d^6y f^2 \sqrt{g} \phi_n(y)^* \left(f^{-1} k^2 \phi_n(y) + \frac{1}{\sqrt{g} f^2} \partial_i g^{ij} f^2 \sqrt{g} \partial_j \phi_n(y) \right), \quad (4.1)$$

and we choose ϕ_n to be eigenfunctions of the operator inside the parenthesis (which is the full Laplacian on the space). The main point is that the 4 dimensional momentum, k , appears explicitly in the eigenfunctions and eigenvalues on the 6 dimensional manifold - the space-time momentum dependence of the action need no longer be quadratic¹³ in momentum once we go to the basis of functions $\phi_{n,k}$.

The off-shell states in the low energy effective action can be derived by a KK reduction, which we will discuss in a minute. However, since the conformal field theory on $SL(2, R)$

¹³ Some non-quadratic actions are intimately related to new symmetries. The familiar example is that of the DBI action [41][42].

was extensively studied over the last few years we can guess the results. Generally in String theory the off-shell effective action is not well defined, since String theory allows only the computations of on-shell quantities. But it does provide a natural guess as to the off-shell modes of some fields in some cases. In the large M_s limit, relaxing the equation of motion in space-time is equivalent to relaxing the constraint that $L_0 + \tilde{L}_0 = 2$ on the operators in String theory. Hence as off-shell states we will take all operators in the conformal field theory that satisfy all physical state conditions, with the exception that we relax this condition.

The spectrum of relevant representations is given in [26]. We will focus on the spectrum of unflowed representations since we will work in the point-particle limit, below the “long strings” threshold [24][25]. We will denote the generators of $SL(2, R)$ by J^\pm and J^3 , where J^3 corresponds to time translations (we are working with global AdS_3 and hence its spectrum is continuous). There are 2 types of relevant representations of $SL(2, R) \times SL(2, R)$:

1. A product of principal discrete representation for the left and right movers $\mathcal{D}_j^\pm \times \mathcal{D}_j^\pm$ with $j > \frac{1}{2}$. The spectrum of these representations satisfies

$$\pm w = 1 + \sqrt{1 + q^2 + m^2 - \lambda} + n_l + n_r, \quad (4.2)$$

where λ is the eigenvalue of the total Laplacian (with the mass term) on $AdS_3 \times T^3$, and we are interested as λ as function of w and q . In this notation $j = \frac{1}{2}(1 + \sqrt{1 + p^2 + m^2 - \lambda^2})$.

2. A product of principal continuous representations $\mathcal{C}_{j=\frac{1}{2}+is}^\alpha \times \mathcal{C}_{j=\frac{1}{2}+is}^\alpha$. In these representations the eigenvalues of the AdS_3 Laplacian and J_3 are uncorrelated. We will mix the representations by writing all of them together as $|j, w_l, w_r\rangle$, $w_l, w_r > 0$, $j = \frac{1}{2} + is$. These representations do not contribute to on-shell degrees of freedom.

The quadratic low energy effective action now becomes

$$S_{off-shell} = S_{off-shell,discrete} + S_{off-shell,continuous} \quad (4.3)$$

where

$$S_{off-shell,discrete} = \sum_{n_l, n_r \geq 0} \int_{|w| > 1 + n_l + n_r} dw \int d^3q \quad (4.4)$$

$$\phi_{w,q,n_l,n_r}^* \left((|w| - 1 - n_l - n_r)^2 - q^2 - m^2 \right) \phi_{w,q,n_l,n_r}$$

and $S_{off-shell,cont.}$ is a higher dimensional integral over w, k and the additional continuous parameters in item 2 above. The details of the latter do not matter for us, except that there is no zero eigenvalue of the Laplacian in this sector.

The three interesting features that we see in this off-shell action are:

1. The discrete states are located on lines $\lambda(w)$ (for fixed q) satisfying (4.2). A 3+1 observer will associate such a line with the off-shell states of a single 3+1 particle (with the mass shell condition being $\lambda = 0$). Such an observer will report that we can define the off-shell modes of a particle only above a certain frequency. Hence we do not have off-shell Green's functions for the 3+1 fields for arbitrary position or momentum. In particular the zero frequency mode was removed, even in the off-shell action.
2. The action is non-analytic in the frequency w . Correspondingly, when Fourier transforming the action for the discrete modes into position space we obtain a non-local action.
3. A continuum appears, with no physical degrees of freedom¹⁴.

It is straightforward to obtain the same results by an ordinary Kaluza-Klein reduction. This is nothing but an exercise in expanding the functions on $AdS_3 \times T^3$ in a complete set of states (the two sets of representations are a complete set of states for the ordinary L^2 norm). A discussion of the discrete representations essentially appears in [36] (and references therein), with the slight modification that there one was looking for on-shell states (and also p^2 was not differentiated from m^2), and hence one sets $\lambda = 0$. This gives a relation between m^2 and w . However, since a non-zero λ is the same as a different m , to go to our case one simply needs to replace m^2 there by $p^2 + m^2 - \lambda$.

It is also easy to see where the continuous representations come from. We will follow the notation in [36] where the metric is

$$ds^2 = \frac{-dt^2 + d\rho^2}{\cos^2(\rho)} + \tan^2(\rho)d\theta^2$$

and the boundary of AdS is at $\rho = \pi$, and we will denote $z = \cos(\rho)$. To have a well-posed problem we will work with a finite cut-off at some finite small value of $z = \epsilon$. At the end of the computations we will take ϵ to 0. We need to impose some boundary condition at

¹⁴ A perhaps more familiar appearance of a path integral with no physical degrees of freedom is in open String field theory after the condensation of the tachyon. There one still writes an action for the open string, but there are no open string physical degrees of freedom.

$z = \epsilon$. The details of the boundary condition will not be important, but for concreteness we can use the boundary conditions in [16] $(z\partial_z - 1 - \sqrt{1 + q^2 + m^2})\Phi|_{z=\epsilon} = 0$. Denoting $\nu = \sqrt{1 + q^2 + m^2 - \lambda}$, the small z behavior of the wave function is [36]

$$\begin{aligned}
& \frac{\Gamma(-\nu)}{\Gamma(\frac{1}{2}(1 - \nu + l + w))\Gamma(\frac{1}{2}(1 - \nu + l - w))}^* \tag{4.5} \\
& *z^{1+\nu} F\left(\frac{1}{2}(1 + \nu + l + w), \frac{1}{2}(1 - \nu + l - w); 1 + \nu; z^2\right) + \\
& + \frac{\Gamma(\nu)}{\Gamma(\frac{1}{2}(1 + \nu + l + w))\Gamma(\frac{1}{2}(1 + \nu + l - w))}^* \\
& *z^{1-\nu} F\left(\frac{1}{2}(1 - \nu + l + w), \frac{1}{2}(1 - \nu + l - w); 1 + \nu; z^2\right).
\end{aligned}$$

To satisfy the boundary condition we need to balance the 2 terms, which requires the 2 terms to be the same order of magnitude. For fixed real ν , the 1st term nominally decays much faster than the 2nd term when $z \rightarrow 0$, and to remedy this we require that one of the Γ functions in the denominator of the 2nd term diverges - this gives us the discrete representations. To obtain the continuous representation we take ν to be imaginary, which gives us another way of cancelling the 2 terms. For finite ϵ the spectrum would be discrete but it would go to a continuum as $\epsilon \rightarrow 0$.

To conclude this section we would like to comment on the relation between the fact that the model is non-Lorentz invariant and the fact that it has a spectrum generating algebra. The dispersion relation

$$w_{n,q} = f(n_L, n_R) + \sqrt{1 + q^2 + m^2}$$

is an interesting way in which one can combine the two, where the algebra changes n_L and n_R . The velocity of a wave packet $\partial w_{n,q}/\partial q$ is the same for all the particles in the multiplet. Hence we can have a locally acting spectrum generating algebra at the price of introducing non-lorentz invariant dispersion relations of the form (2.6).

5. Generalizations

Uniqueness of the Basic Model

We have discussed so far the background $AdS_3 \times S^3 \times T^4$. We would like to know whether there are other backgrounds to which we can apply our SUSY breaking mechanism.

As a first step we would like to consider all backgrounds of the type $AdS_3 \times T^3 \times \mathcal{N}$, where \mathcal{N} is a compact conformal field theory, such that the space-time theory has at least $(2, 2)$ supersymmetry. We will show that under these circumstances the model actually has $(4, 4)$ supersymmetry. This leaves us with the $AdS_3 \times S_3 \times T^4$ as the main model¹⁵, and models built on this basic example, which we will mention later.

We will use [43], which specifies the conditions under which $AdS_3 \times \mathcal{N}'$ will have $(2, 2)$ supersymmetry. As there $SL(2)$ will be taken at level k , leading to central charge $c^{SL(2)} = \frac{3(k+2)}{k} + \frac{3}{2}$. To have supersymmetry we require that:

1. \mathcal{N} contains an affine $U(1)$ current $\psi^{U(1)} + \theta J^{U(1)}$
2. $\mathcal{N}/U(1)$ is an $N = 2$ superconformal theory with central charge $c^{\mathcal{N}/U(1)} = 9/2 - 6/k$.

We will bosonize its current with a canonically normalized scalar $J_R^{\mathcal{N}/U(1)} = ia\partial z$, where $a = \sqrt{c^{\mathcal{N}/U(1)}/3}$.

The $SL(2)$ is made out of 3 bosonic $SL(2)$ current and 3 free fermions $\Psi^{1,2,3}$. We will define the following bosons

$$\begin{aligned}\partial H_1 &= \Psi^1 \Psi^2 \\ \partial H_2 &= \Psi^3 \Psi^{U(1)} \\ i\sqrt{3}\partial H_0 &= J^{\mathcal{N}/U(1)} - \sqrt{\frac{2}{k}} J^{U(1)}\end{aligned}$$

The space-time susy generators are

$$G_r^\pm \propto \int dz e^{-\phi/2} S_r^\pm \dot{S}, \quad r = \pm \frac{1}{2}$$

where

$$S_r^\pm = e^{ir(H_1 \pm H_2) \pm i(\sqrt{3}/2)H_0}$$

and \dot{S} are the spin operators from the T^3 directions. We end up with the model having at least 2 complex supersymmetries in each of the left and right moving sector. This implies that the model has at least $(4, 4)$ supersymmetry.

More Examples

Once we have constructed the basic example as above, we can modify it in different ways to generate a richer spectrum. Again we would like to concentrate on examples in

¹⁵ There are other models that have $(4, 4)$ supersymmetry, but these reduce to the model above locally, ie., when we go to the regime of parameter space where 3 coordinates become large (in addition to time). For example, $AdS_4 \times S^3 \times K3$.

which the SUSY breaking mechanism described above is still valid. This means that we can do any modification as long as we do not change the boundary CFT, ie, we can do any change that we want in the interior of AdS_3 , which would correspond to turning on states in the CFT, but we cannot deform the behavior near the boundary. This means that we are still discussing the same theory which has the SUSY breaking deformation.

Two kinds of deformations are potentially interesting as they may give solvable CFTs. One is orbifolding the $AdS_3 \times S^3 \times T^4$ in a way which does not act on the time or on the large T^3 . Examples of these were discussed in [44][45][46]. Another modification is to place some branes at the origin of AdS , wrapping the T^3 . The fact that the branes do not intersect the boundary of AdS_3 means that they correspond to states in the same CFT as before.

These branes can have various configurations on the S^3 and on the remaining S^1 . If one generalizes the model both by orbifolding and adding branes to the orbifold, a very rich set of spectra can emerge. Relative to ordinary compactifications we can add branes much more liberally. The reason is that charge conservation is less of an obstacle here since the manifold is topologically non compact and the flux can escape to infinity.

This might also help in terms of the Lorentz invariance of the spectrum. Consider the simplest case of a brane wrapping the time direction and the T^3 and situated at the origin of AdS_3 . The metric that excitations on this brane see is

$$-g_{00}(\rho = 0)dt^2 + dx_i^2,$$

which has a Lorentz symmetry. This symmetry is, of course, not exact because of coupling to non-lorentz invariant background fields, non-lorentz invariant closed strings etc. Still, one expects that Lorentz invariance will be improved in the D-brane sector vs. the closed string sector because the low energy closed strings are states with width of order $1/\Lambda$ on the AdS , whereas low lying open strings are of width l_s around the position of the brane. If we take $l_s < 1/\Lambda$, as we have been doing so far, the smaller width on the AdS will translate into improved Lorentz invariance.

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